Unusual Transition Patterns in Bose-Einstein Condensation

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We analyze the possible transition patterns exhibited by an effective non-relativistic field model describing interacting binary homogeneous dilute Bose gases whose overall potential is repulsive. We evaluate the temperature dependence of all couplings and show that at intermediate temperatures the crossed interaction, which is allowed to be attractive, dominates, leading to smooth re-entrant phases. At higher temperatures this interaction suffers a sudden sign inversion leading to an abrupt discontinuous transition back to the normal gas phase. This situation may suggest an alternative way to observe collapsing and exploding condensates. Our results also suggest that such binary systems may offer the possibility of observing Bose-Einstein condensation at higher critical temperatures.

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The study of symmetry breaking (SB) and symmetry restoration (SR) mechanisms have proven to be extremely useful in the analysis of phenomena related to phase transitions in almost all branches of physics. Some topics of current interest which make extensive use of SB/SR mechanisms are topological defects formation in cosmology, the Higgs-Kibble mechanism in the standard model of elementary particles and the Bose-Einstein condensation (BEC) in condensed matter physics. An almost general rule that arises from those studies is that a symmetry which is broken at zero temperature should get restored as the temperature increases. Examples range from the traditional ferromagnet to the more up to date chiral symmetry breaking/restoration in QCD, with the transition pattern being the simplest one of going from the broken phase to the symmetric one.

A counter-intuitive example may happen in multi-field scalar models, as first noticed by Weinberg [1]. Considering an $O(N) \times O(N)$ invariant relativistic model, with two types of scalar fields and different types of self and crossed interactions, Weinberg has shown that it is possible for the crossed coupling constant to be negative, while the model is still bounded from below, leading, for some parameter values, to an enhanced symmetry breaking effect at high temperatures. This would predict that a symmetry which is broken at $T = 0$ may not get restored at high temperatures, a phenomenon known as symmetry nonrestoration (SNR), or, in the opposite case, a symmetry that is unbroken at $T = 0$ would become broken at high temperatures, a case called inverse symmetry breaking (ISB). Since then the model has been re-investigated by many other authors using a variety of different methods, both perturbative and nonperturbative analytical and numerical methods, giving further support to the idea of SNR/ISB. A recent review [2] lists most applications and gives an introduction to the subject, discussing other contexts in which SNR/ISB can take place in connection with cosmology and condensed matter physics.

Two of the present authors have also treated the problem nonperturbatively taking full account of the cumbersome two-loop contributions [3]. The results obtained in Ref. [3] were shown to be in good agreement with those of Ref. [4], where the SNR/ISB phenomena were studied using the Wilson Renormalization Group (WRG) and the explicit running of the (temperature) dependent coupling constants has been taken into account, showing that in fact the strength of all couplings increase with the temperature, enhancing SNR/ISB. This result completely rules out a possible decrease of the strength of the negative crossed coupling that would lead to the eventual SR at higher temperatures. These interesting results from finite temperature quantum field theory raise important questions regarding a possible experimental observation of those phenomena in current laboratory experiments.

The recent experimental achievement of BEC and its further improvements has opened the interesting possibility of probing and studying finite temperature quantum field models and methods, currently used in cosmology and particle physics, in the laboratory. One of the remarkable things about BEC of dilute atomic gases is the possibility of adjusting several experimental features by fine-tuning the parameters with a high level of control and accuracy (for recent reviews see Ref. [5]). We could, for instance, envisage the possibility of investigating the SNR/ISB phenomena using a system composed by a mixture of coupled atomic gases, like the ones recently produced [6] in which one has the same chemical element in two different hyperfine states and that may be treated as "effectively distinguishable", or just consider the mixing of two different mono-atomic Bose gases.

Here, we shall do a theoretical investigation of the possible transition patterns followed by this type of system paying special attention to the SNR issue since one expects, on basic grounds, that as the temperature increases the atoms will leave the condensate phase going to the symmetric phase of a normal gas. As a byproduct we shall see how the crossed couplings shift the critical temperatures with respect to those obtained for uncoupled gases, creating the possibility of obtaining BEC at higher temperatures.

The model we consider is similar to the ones used in other theoretical studies of homogeneous dilute coupled

Bose gases [7], that consists of a hard core sphere gas model described by non-relativistic interacting (complex) scalar fields, with an overall repulsive potential. This system can be described by the following $U_\psi(1)\times U_\phi(1)$ invariant finite temperature Euclidean spacetime action

$$S_E(\beta)=\int_0^\beta d\tau\int d^3x\left[\psi^*\left(\frac{\partial}{\partial\tau}-\frac{\nabla^2}{2m_\psi}-\mu_\psi\right)\psi+\frac{g_\psi}{4}(\psi^*\psi)^2 + \phi^*\left(\frac{\partial}{\partial\tau}-\frac{\nabla^2}{2m_\phi}-\mu_\phi\right)\phi+\frac{g_\phi}{4}(\phi^*\phi)^2+g(\psi^*\psi)(\phi^*\phi)\right], \quad (1)$$

where, in natural unities, $T=1/\beta$. The associated chemical potentials are represented by μ_i ($i=\psi$ or ϕ) while m_i represent the masses. For the hard core sphere self-interactions we take the phenomenological coupling constants as being the ones normally used in the absence of crossed interactions and which are valid in the dilute gas approximation [5]. In terms of the corresponding s-wave scattering lengths a_i they can be written as $g_i=8\pi a_i/m_i$. To make contact with the analogous potential used in the prototype relativistic models for SNR, we take the overall potential as being repulsive, bounded from below. This requirement imposes the constraint condition $g_\psi>0$, $g_\phi>0$ and $g_\psi g_\phi>4g^2$.

At $T=0$, in the absence of crossed interactions and for $\mu_i>0$ the classical potential of Eq. (1) exactly reproduces the case of an uncoupled gas with symmetry broken ground states representing "condensates" $|\psi|=(2\mu_\psi/g_\psi)^{1/2}$ and $|\phi|=(2\mu_\phi/g_\phi)^{1/2}$. At the same time, the case $\mu_\psi<0$ and $\mu_\phi<0$ corresponds to the normal symmetric phases without any condensates, $|\psi|=0$ and $|\phi|=0$. Our aim is to investigate, for non-negligible binary interactions, how the inclusion of temperature corrections can alter this picture. This can be achieved by computing the temperature dependent effective chemical potentials (see for instance Ref. [8] for an analogous analysis for the mono-atomic gas) defined as a solution of the gap equation $\mu_i(T)=\mu_i(0)+\Sigma_i^T(\mathbf{p})$, where $\Sigma_i^T(\mathbf{p})$ is the field temperature dependent self-energy.

The phase structure of the model is then given by the sign of $\mu_i(T)$ at a given temperature. One has $\mu_i(T<T_c^i)>0$ in the broken condensate phase and $\mu_i(T>T_c^i)<0$ in the symmetric normal-gas phase. At the same time the Hugenholtz-Pines theorem imposes $\mu_i(T=T_c^i)=0$. Our next step is to evaluate the thermal self-energies which we shall do in a nonperturbative self-consistent fashion so as to avoid any potential problems associated with the usual perturbative calculation (see, *e.g.*, Ref. [3] for a discussion in relativistic field theory). One can perform a one-loop self-consistent resummation by using the effective dressed propagator $D_{i,i^*}(\mathbf{p},\omega_n)=[-i\omega_n+\omega_i]^{-1}$ where $\omega_n=2\pi n/\beta$ are the bosonic Matsubara frequencies and $\omega_i=\mathbf{p}^2/(2m_i)-\mu_i(T)$. One gets

$$\Sigma_i^T(\mathbf{p})=-\frac{1}{\beta}\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left[\frac{g_i(0)}{-i\omega_n+\omega_i}+\frac{g(0)}{-i\omega_n+\omega_j}\right], \quad (2)$$

where $g_i(0)$ and $g(0)$ indicate the bare, zero temperature, coupling constants. The sum in Eq. (2) can be easily performed and the resulting momentum integrals lead to well known Bose integrals [9]. For simplicity, in the following we consider atoms (fields) with approximately the same mass, $m_\psi\simeq m_\phi=m$. One obtains two one-loop self-consistent coupled equations given by

$$\mu_i(T) = \mu_i(0)-g_i(0)\left(\frac{m}{2\pi\beta}\right)^{3/2}\mathrm{Li}_{3/2}[\exp(\beta\mu_i(T))] - g(0)\left(\frac{m}{2\pi\beta}\right)^{3/2}\mathrm{Li}_{3/2}[\exp(\beta\mu_j(T))], \quad (3)$$

where $\mathrm{Li}_n(z)$ is the polylogarithmic function. As shown below, $\mu_i(0)<<T_c$ for realistic parameter values. We shall also see that $\mu_i(T)$ quickly decreases (in the SR case) or remains approximately equal to $\mu_i(0)$ (in the SNR case) with increasing T so that one may safely consider the high temperature approximation $\mu_i(T)/T<<1$ by taking $\mathrm{Li}_{3/2}[\exp(\beta\mu_j(T))]\sim\zeta(3/2)$. One then obtains the critical temperatures

$$T_c^i=\left(\frac{2\pi}{m}\right)\left\{\frac{\mu_i(0)}{[g_i(0)+g(0)]\zeta(3/2)}\right\}^{2/3}. \quad (4)$$

Eq. (4) displays some unusual effects due to the presence of crossed interactions. There are three interesting cases which depend on the sign and magnitude of $g(0)$. Taking $g(0)>0$ one observes a shift in the critical temperatures indicating that the BEC/normal-gas transition occurs at lower temperatures compared to the usual mono-atomic case ($g(0)=0$). If $g(0)<0$ but $|g(0)|<g_i(0)$ then the transition occurs at higher temperatures. Despite these important quantitative differences symmetry restoration does take place in both cases. Now consider the case where $g(0)<0$ but $|g(0)|>g_\phi(0)$ (in this case the boundness condition assures that $|g(0)|<g_\psi(0)$). For this situation something unexpected occurs concerning the ϕ field since Eq. (4) does not allow for a finite, positive real critical temperature value. This is a manifestation of SNR within our two-field model and is analogous to what is seen in the relativistic case. At the same time the field ψ suffers the expected phase transition at a higher T_c compared to the $g(0)=0$ case. Obviously, which field will suffer SNR depends on our initial choice of couplings.

For the following analysis let us set the parameters by choosing representative values for a mixture of gases such as ^{85}Rb and ^{87}Rb. Some realistic values are $m=86$GeV and $a_\psi=2.5\times10^{-2}(\mathrm{eV})^{-1}$, corresponding to the s-wave scattering length of ^{87}Rb, which fix the coupling $g_\psi(0)=7.3(\mathrm{MeV})^{-2}$. Setting $\mu_\psi(0)=5.67$peV the value $T_c\simeq32.5$peV $=280$nK is reproduced in the usual $g=0$ case. Throughout this Letter we set $\mu_\phi(0)=\mu_\psi(0)$, keep $g_\psi(0)$ fixed, while considering $g_\phi(0)$ and $g(0)$ as tunable parameters. In principle, this can be experimentally achieved by appropriately setting magnetic fields close to

a Feshbach resonance as described in a recent application to ^{85}Rb [10]. As an illustration of the relevant phenomena we want to describe, we consider three possible sets of numerical values for $g_\phi(0)$ and $g(0)$, given, in unities of MeV^{-2} by: I) $g_\phi(0) = 5.6$ and $g(0) = 2.3$; II) $g_\phi(0) = 5.6$ and $g(0) = -2.3$ and III) $g_\phi(0) = 0.56$ and $g(0) = -0.84$. Based on the general results given by Eqs. (3) and (4) we then have the usual transition patterns for both ψ and ϕ for sets I and II where $\mu_\psi(T)$ and $\mu_\phi(T)$ go from positive to negative values, with transition temperatures that can be easily derived from Eq. (4). For set III, the field ψ exhibits the usual transition from the condensed to the normal gas phase as the temperature is increased from zero. However, we have $\mu_\phi(T) > 0$ for arbitrarily large temperatures, which corresponds to SNR. This result is however misleading, as we demonstrate below.

As a general result from the renormalization group, we expect that all coupling constants should run with the temperature [4]. For instance, if $g_i(T)$ decreases faster than $g(T)$, or on the other way around, if the crossed coupling decreases faster than the self-couplings, the simple analysis performed above can change drastically. As noted in the introduction, those substantial qualitative changes were not observed in the relativistic case. However, a basic difference, in between the relativistic and non-relativistic model studied here, refers to the type of four-point functions allowed by each model. In fact, the contributions considered in the relativistic calculations include the t and u scattering channels as well as the s-channel contributions. At the same time, elastic and inelastic collisions are allowed whereas for the present case only elastic, s-channel contributions can be considered since our effective model represents a system of hard core spheres. Keeping these facts in mind and performing a nonperturbative one-loop calculation in terms of temperature dependent vertices and effective propagators one obtains

$$g(T) = g(0)\left[1 + g(0)B_{i,j}(\mathbf{k})\right]^{-1}, \tag{5}$$

where $B_{i,j}(\mathbf{k})$ represents the bubble contribution,

$$B_{i,j}(\mathbf{k}) = s_{i,j}\,\beta\int_0^1 d\alpha\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{\exp(\beta\Omega_{i,j})}{\left[\exp(\beta\Omega_{i,j})-1\right]^2}\,. \tag{6}$$

The quantity $s_{i,j}$ is a symmetry factor, $s_{i,j} = 2$ for $i \neq j$ and $s_{i,j} = 5/2$ for $i = j$, while $\Omega_{i,j}$ is given by

$$\Omega_{i,j} = \frac{\mathbf{q}^2 + \alpha(1-\alpha)\mathbf{k}^2}{2m} - \alpha\mu_i(T) - (1-\alpha)\mu_j(T)\,, \tag{7}$$

where $\mathbf{k}$ represents the s-channel incoming momentum and α is a Feynman parameter introduced to merge the two internal bubble propagators in the derivation of Eq. (6). The $\mathbf{q}$ integral in Eq. (6) is again performed with the help of the Bose integrals and one obtains

$$B_{i,j}(\mathbf{k}) = s_{i,j}\beta\left(\frac{m}{2\pi\beta}\right)^{3/2}\int_0^1 d\alpha\;\mathrm{Li}_{1/2}[\exp(-\beta\chi_{i,j})], \tag{8}$$

where $\chi_{i,j} = \left[\alpha(1-\alpha)\mathbf{k}^2/(2m) - \mu_i(T)\alpha - \mu_j(T)(1-\alpha)\right]$. Using the expansion $\mathrm{Li}_{1/2}[\exp(-z)] = 1.77/z^{1/2} - 1.46 + 0.2082z - 0.0128z^2 + \mathcal{O}(z^3)$ (see Ref. [9]) and setting $\mathbf{k}^2/2m = 3T$ (the average incoming two particle kinetic energy), the α integral in Eq. (8) can be performed. In the high temperature limit, one obtains

$$g(T) \simeq g(0)\left[1 + \gamma_{i,j}\;g(0)\;T^{1/2}\left(\frac{m}{2\pi}\right)^{3/2}\right]^{-1}, \tag{9}$$

where $\gamma_{i,j} = 3.7012$. The equations for the couplings representing the self-interactions, $g_\psi(T)$ and $g_\phi(T)$, have exactly the same structure and can be obtained by replacing g with g_ψ or g_ϕ in Eq. (9) and $\gamma_{i,j}$ by $\gamma_{i,i} = 4.6265$. These results show that $g_\psi(T)$ and $g_\phi(T)$ always decrease, approaching zero monotonically at high temperatures since $g_\psi(0)$ and $g_\phi(0)$ are positive quantities. The same happens for $g(T)$, if $g(0)$ is positive. However, for $g(0) < 0$, it can be easily checked that $g_\psi(T)$ and $g_\phi(T)$ decrease faster than $g(T)$. Consequently the transition picture described above will be shown to change in an unexpected way. In this case, Eq. (9) shows that $g(T)$ is initially negative with an absolute which increases up to a certain critical inversion temperature given by $T_g = (2\pi/m)^3[3.7012\;g(0)]^{-2}$, where it develops a pole. Then, for $T > T_g$, $g(T)$ suddenly becomes positive which means that, contrary to the relativistic case, one always has symmetry restoration at high temperatures in the nonrelativistic model of hard core spheres. It is important to note that this pole in Eq. (9) has a completely different origin from the usual Landau pole found in the relativistic ϕ^4 theory. Here the pole signals a phase transition and if the nonperturbative self-consistent loop expansion used here converges, as generically expected, a next order calculation will only lead to a small change in the value of T_g.

It is instructive to look in more detail at sets II and III where $g(0)$ is negative. The results for $\mu_i(T)$ are shown in Fig. 1. Contrary to the previous analysis, with bare (temperature independent) coupling constants, we now have a completely unexpected behavior for both fields for some parameters values, as the ones exemplified by sets II and III. For set III, Fig. 1 shows that ϕ displays the typical SNR behavior while ψ undergoes SR. The same figure shows that, for set II, both fields observe SR. In the three SR cases above, the transition from the BEC phase to the normal-gas phase is observed at critical temperatures between 380nK and 900nK. Then, as soon as the negative $g(T)$ reaches a magnitude which is larger than that of $g_i(T)$, both type of atoms return to the BEC phase in a re-entrant transition. We note that, for set III, the field ψ also goes through a re-entrant phase

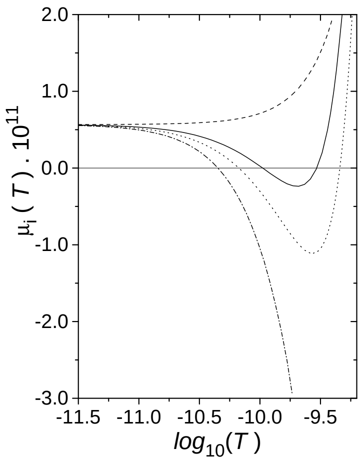

FIG. 1: The effective chemical potentials $\mu_\psi(T)$ (dotted and dot-dashed curves, for sets II and III, respectively) and $\mu_\phi(T)$ (continuous and dashed curves, for sets II and III, respectively) as a function of temperature (in units of eV).

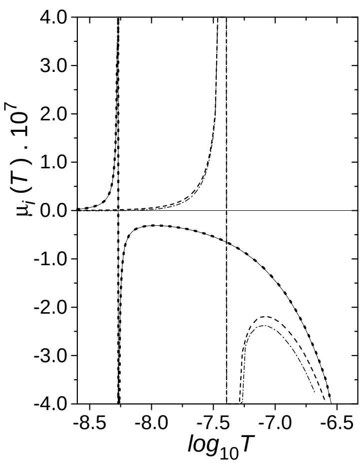

FIG. 2: The high temperature discontinuous transitions suffered by both fields. Same notation as in Fig. 1.

that happens at a higher temperature scale $\sim 50\mu$K, not shown in Fig. 1.

At the intermediate temperatures the boundness condition $g_\psi(T)g_\phi(T) > 4g(T)^2$ does not hold, the overall potential becomes highly attractive and unbounded from below. Then, suddenly, at the inversion temperature $T = T_g$ the potential becomes highly repulsive and both type of atoms leave the ground state at once. Contrary to the first BEC/normal-gas transition, this second passage to the gas phase is a discontinuous transition due to the sudden sign inversion of $g(T)$. This transition is shown in Fig. 2 for cases II and III for both ψ and ϕ. For set II both fields go to the gas phase at a critical temperature $T_g \sim 40\mu$K, while for set III, $T_g \sim 340\mu$K. These values should be contrasted to the ones for the critical temperatures for the uncoupled gases, of $T_c^\psi \sim 280$nK, for both cases and $T_c^\phi \sim 330$nK and ~ 1800nK, for cases II and III, respectively.

Here, we cannot furnish more details concerning this discontinuous transition since, at the transition, our high temperature approximation and the effective model of two body interactions are not valid due to the very large values acquired by the $\mu_i(T)$ and $g(T)$. However, this should not invalidate the qualitative features of the transition, at least on its neighborhood.

In conclusion our results show that a system of coupled Bose gases with an overall repulsive potential, but attractive crossed interactions, may display unusual transition patterns, when the important temperature dependence of the couplings are taken into account. These patterns include the usual continuous transition condensate/normal-gas followed, at intermediate temperatures, by an unexpected re-entrant, continuous, normal-gas/condensate transition. Higher temperatures induce a sudden change in the sign of the crossed coupling followed by a dramatic discontinuous transition of the type condensate/normal-gas. Those phases suggest a possible pattern for the observation of collapsing and exploding condensates. It is important to note that experimentally one knows that a change on the sign of the couplings may be achieved by adjusting external magnetic fields [10]. However, our results further indicate that, at least for coupled systems, the temperature can also act as the external agent which drives the sign inversion. In addition, our results explicitly demonstrate that, contrary to the relativistic case, and according to intuition, SNR cannot happen in the non-relativistic model of hard core spheres with temperature dependent couplings. Finally, the coupled Bose-Einstein gas system with attractive crossed interaction also seems to offer an avenue to obtain condensation at higher temperatures.

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